ISSN 2229-5518

A novel fast version of particle swarm optimization method applied to the problem of optimal capacitor placement in radial distribution systems

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Abstract— Particle swarm optimization (PSO) is a popular and robust strategy for optimization problems. One main difficulty in applying PSO to real-world applications is that PSO usually needs a large number of fitness evaluations before a satisfying result can be obtained. This paper presents a modified version of PSO method that can converge to the optima with less function evaluation than standard PSO. The main idea is inserting two additional terms to the particles velocity expression. In any iteration, the value of the objective function is a criterion presenting the relative improvement of current movement with respect to the previous one. Therefore, the difference between the values of the objective function in subsequent iterations can be added to velocity of particles, interpreted as the particle acceleration. By this modification, the convergence becomes fast due to new adaptive step sizes. This new version of PSO is called Fast PSO (FPSO). To evaluate the efficiency of FPSO, a set of benchmark functions are employed, and an optimal capacitor selection and placement problem in radial distribution systems is evaluated in order to minimize cost of the equipment, installation and power loss under the additional constraints. The results show the efficiency and superiority of FPSO method rather than standard PSO and genetic algorithm.

Index Terms— convergence speed, fast PSO, capacitor placement, particle swarm optimization, radial distribution system

1 Introduction

In recent years, many optimization algorithms are introduced. Some of these algorithms are traditional optimization algorithms. Traditional optimization algorithms use exact methods to find the best solution. The idea is that if a problem can be solved, then the algorithm should find the global best solution. Large search spaces increases the evaluation cost of these algorithms. Therefore the complex large spaces slow down the convergence rate of the algorithm to find the global optimum. Linear and nonlinear programming, brute force or exhaustive search and the divide and conquer methods are some of the exact optimization methods.

Other optimization algorithms are stochastic algorithms, consisted of intelligent, heuristic and random methods. Stochastic algorithms have several advantages compared to other algorithms as follows [1]:

1. Stochastic algorithms are generally easy to implement.
2. They can be used efficiently in a multiprocessor environment.
3. They do not require the problem definition function to be continuous.
4. They generally find optimal or near-optimal solutions.

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Two well known intelligent stochastic algorithms are Particle swarm optimization (PSO) and genetic algorithm (GA). PSO is a population-based searching technique proposed in 1995 [2] as an alternative to GA [3]. Its development is based on the observations of social behavior of animals such as bird flocking, fish schooling, and swarm theory. Compared with GA, PSO has some attractive characteristics. First of all, PSO has memory, that is, the knowledge of good solutions is retained by all particles, whereas in GA, previous knowledge of the problem is destroyed ones the population is changed. Secondly, PSO has constructive cooperation between particles, i.e. particles in the swarm share their information.

In today's power system, there is a trend to use nonlinear loads such as energy-efficient fluorescent lamps and solid-state devices. Distribution systems provide power to a wide variety of load types. Resistive loads (power factor = 1.0) require no reactive power at all, while induc-

ISSN 2229-5518

tive loads (power factor < 1.0) require both active and reactive power. Inductive loads (e.g. motors) are always present so that the line current consists of a real (or resistive) component and an inductive component. Both components contribute to the MW losses (which are proportional to the square of the current magnitude), voltage drop and line loading (measured in A or MVA) reactive currents increased ratings for distribution components. The resistive component of the current cannot be substantially reduced as this is the part of the current that actually performs work (defined by demand). The reactive component of the current can be reduced by installation of capacitors "close" to the loads. This has the effect that the reactive power needed is generated locally and the distribution circuits are relieved from the reactive power transfer. Effective placement of the shunt capacitors (depending on the situation) can improve the voltage profile and can greatly reduce the losses and the line loading. The capacitor sizing and allocation should be properly considered, or else they can amplify harmonic currents and voltages due to possible resonance at one or several harmonic frequencies. This condition could lead to potentially dangerous magnitudes of harmonic signals, additional stress on equipment insulation, increased capacitor failure and interference with communication system. Thus, the problem of optimal capacitor placement consists of determining the locations, sizes, and number of capacitors to install in a distribution system such that the maximum benefits are achieved while operational constraints at different loading levels are satisfied [4].

Optimal capacitor placement has been investigated since the 60's. Early approaches were based on heuristic techniques applied to relaxed versions of the problem (some of the more difficult constraints were dropped). In the 80's, more rigorous approaches were suggested as illustrated by the paper by Grainger [5], [6]. Baran-Wu [7] have formulated the capacitor placement problem as a mixed integer nonlinear program: the problem then has been approximated by a differentiable function that allowed the solution by Benders decomposition. In the 90's, combinatorial algorithms were introduced as a means of solving the capacitor placement problem: simulated annealing has been proposed in [8], genetic algorithms in [9], and tabu search algorithms in [10]. Delfanti et. al. [11] have introduced a genetic algorithm (GA) approach in VAR planning of a small CIGRE system of the Italian transmission and distribution network in order to determine the minimum investment required to satisfy suitable reactive power constraints. Unfortunately the introduced GA algorithm had the problem of a large number of simplex iterations leading to very long computation time.

However, in this paper, a modified PSO, named fast PSO (FPSO) is proposed which asserts fast convergence property and consequently lower the number of function evaluation. FPSO possesses two additional terms added to the standard PSO velocity updating formula. These statements cause FPSO to move to the optimal solution faster than the standard PSO, adaptively. Therefore, these modifications speed up the PSO convergence rate. The effectiveness and efficiency of the proposed FPSO is first examined using some well-known optimization benchmarks. Then, it is applied to the problem of optimal capacitor placement in radial distribution systems. Numerical simulations are presented to validate the applicability and efficiency of our modified optimization scheme.

The rest of this paper is preceded as follows. Section 2 presents the standard PSO algorithm. Section 3 introduces FPSO. In Section 4, first the efficiency of FPSO algorithm is verified using some standard test functions. Then, the problem of optimal capacitor placement is solved using proposed FPSO method. Finally, some conclusions are given in section 5.

2 The standard PSO algorithm

2.1 Review Stage

A particle swarm optimizer is a population based stochastic optimization algorithm modeled based on the simulation of the social behavior of bird flocks. PSO is a population-based search process where individuals initialized with a population of random solutions, referred to as particles, are grouped into a swarm. Each particle in the swarm represents a candidate solution to the optimization problem, and if the solution is made up of a set of variables, the particle can correspondingly be a vector of variables. In PSO system, each particle is "flown" through the multidimensional search space, adjusting its position in the search space according to its own experience and that of neighboring particles. The particle, therefore, makes use of the best position encountered by itself and that of its neighbors to position itself toward an optimal solution. The performance of each particle is evaluated using a predefined fitness function, which encapsulates the characteristics of the optimization problem [12].

The core operation of PSO is the updating formulae of the particles, i.e. the equation of velocity updating and the equation of position updating. The global optimizing model proposed by Shi and Eberhart is as follows [12]:

$$v_{i+1}=w\times v_i+RAND\times c_1\times(P_{best}-x_i)+rand\times c_2\times(G_{best}-x_i) \quad (1)$$

$$x_{i+1}=x_i+v_{i+1} \quad (2)$$

where w is the inertia weight factor, vi is the velocity of particle i, xi is the particle position and C_1 and C_2 are two positive constant parameters called acceleration coefficients. RAND and rand are the random functions in the range [0, 1], Pbest is the best position of the ith particle and Gbest is the best position among all particles in the swarm.

3 Fast PSO algorithm

The main drawback of PSO approach is slow convergence specifically prior to providing an accurate solution, closely related to its lack of any adaptive acceleration terms in the velocity updating formulae. In equation 1, c1 and c2 determine the step size of the particles movements through the Pbest and Gbest positions, respectively. In the original PSO, these step sizes are constant and for the all particles are the same.

In any iteration, the value of the objective function is a criterion that presents the relative improvement of this movement with respect to the previous one. Thus the difference between the values of the objective function in consequent iterations can represent the particle acceleration.

Therefore, velocity updating formulae turns to the following form.

$$v_{i+1} = w\times v_i + (f(P_{best}) - f(x_i)) \times RAND\times c_1 \times (P_{best} - x_i) + (f(G_{best}) - f(x_i)) \times rand \times c_2 \times (G_{best} - x_i) \quad (3)$$

where $f(P_{best})$ is the best fitness function that is found by i[th] particle and $f(G_{best})$ is the best fitness function that is found by swarm up to now and other parameters are chosen the same as section A. These two terms, i.e. $(f(P_{best}) - f(x_i))$ and $(f(G_{best}) - f(x_i))$ cause to have an adaptive movement.

The steps of FPSO algorithm are as follows:

1. Initialize particles positions and velocities, randomly.
2. Calculate the objective functions values of particles.
3. Update the global and local best positions and their objective function values.
4. Calculate new velocities using equation (3).
5. Update the positions.
6. If stop condition is attained stop otherwise go to step 2.

Remarks:

1. *The term* $(f(P_{best}) - f(x_i))$ *and* $(f(G_{best}) - f(x_i))$ *are named local and global adaptive coefficients, respectively. In any iteration, the former term defines the movement step size in the best position's direction which is found by ith particle, and the later term defines adaptive movement step size in the best optimum's direction point which ever have been found by the swarm. In other words, the adaptive coefficients decrease or increase the movement step size relative to being close or far from the optimum point, respectively. By means of this method, velocity can be updated adaptively instead of being fixed or changed linearly. Therefore, using the adaptive coefficients, the convergence rate of the algorithm will be increased rather than performed by the proportional large or short steps.*
2. *Stochastic optimization approaches suffer from the problem of dependent performance. This dependency is usually because of parameter initializing. Thus, we expect large variances in performance with regard to different parameter settings for FPSO algorithm. In general, no single parameter setting exists which can be applied to all problems. Therefore, all parameters of FPSO should be determined optimally, by trial and error.*
3. *There are three stopping criteria. The first one is related to the maximum number of allowable iterations set for the algorithm. The second one is when no improvement has been made for a certain number of iterations in the best solution, and the third one is when a satisfactory solution is found.*
4. *The fast version of PSO is proposed for continuous variable functions. Moreover, the main idea of speeding up can be applied to the discrete form of the PSO [13]. We take this into our consideration as a future work.*
5. *Increasing the value of the inertia weight, w, would increase the speed of the particles resulting in more exploration (global search) and less exploitation (local search). On the other hand, decreasing the value of w will decrease the speed of the particle resulting in more exploitation and less exploration. Thus, an iteration-dependent weight factor often outperforms a fixed factor. The most common functional form for this weight factor is linear, and changes with step i as follows:*

$$w_{i+1} = w_{max} - \frac{w_{max} - w_{min}}{N_{iter}} \quad (4)$$

where N_{iter} *is the maximum number of iterations and* W_{max} *and* W_{min} *are selected to be 0.8 and 0.2, respectively.*

6. *Lastly, the proposed FPSO is still a general optimization algorithm that can be applied to any real world continuous optimization problems. In the next section, we will apply FPSO approach to several benchmark functions and compare the results with standard PSO and GA algorithms.*

4 Results of Simulations

In this section, first the efficiency of FPSO is tested using a set of test functions. Then, the FPSO algorithm is applied to solve the problem of optimal capacitor placement.

4.1 Evaluating FPSO using benchmarks

Here, some well-known benchmark functions (listed in Appendix A) are used to examine the effectiveness and convergence speed of the proposed FPSO technique. To avoid any misinterpretation of the optimization results, related to the choice of any particular initial populations, we performed each test 100 times, starting from various randomly selected solutions, inside the hyper rectangular search domain specified in the usual literature.

The performance of FPSO is compared to continuous GA and PSO algorithms using 15 functions listed in Appendix A. The experimental results obtained for the test functions, using the 3 mentioned different methods, are given in Table 1. In our simulations, each population in GA has 10 chromosomes and a swarm in FPSO and PSO has 10 particles. Other parameters of the three algorithms are selected optimally by trial and error. For each function, we give the average number of function evaluations for 100 runs. The best solution found by 3 methods was similar, so there are not given here.

Note that the FPSO has shown drastically better results in convergence and evaluation costs compared with GA and PSO, as it utilizes adaptive movements to reach to the optima, while GA and PSO do not have such an element. Fig. 1 shows a typical diagram of three algorithms convergence rates for B2 function, starting from a same initialization point. As it can be seen, although, all algorithms can find the optima, but FPSO is dramatically faster than the others. Therefore, in many real world applications where real time computations and less CPU time consumption are necessary, FPSO may work better than GA and PSO.

TABLE 1

AVERAGE NUMBER OF OBJECTIVE FUNCTION EVALUATIONS USED BY THREE METHODS

Function/Method	FPSO	PSO	GA
RC	38	42	48
ES	67	90	92
GP	31	39	41
B2	27	33	32
SH	44	61	57
R_2	45	63	65
Z_2	50	62	62
DJ	38	55	60
$H_{3,4}$	39	68	71
$S_{4,5}$	55	82	91
$S_{4,7}$	51	74	76
$S_{4,10}$	53	81	79
R_5	159	244	251
Z_5	123	194	171
$H_{6,4}$	132	212	195

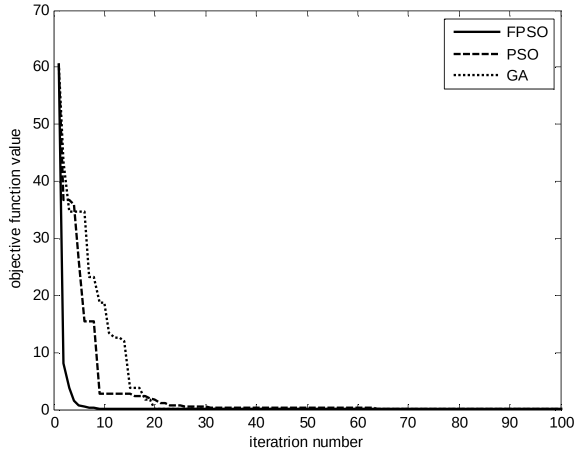

Fig. 1. Convergence rate of three algorithms for B2 function

5 SOLVING OPTIMAL CAPACITOR PLACEMENT USING FPSO

Now that the efficiency and high speed convergence property of the proposed FPSO algorithm has been revealed by simulation results of benchmark functions, the next step is to solve the problem of optimal capacitor placement with FPSO method.

The optimal capacitor placement problem has many variables including the capacitor size, capacitor location and capacitor equipment and installation costs. In this section we consider a distribution system with nine possible locations for capacitors and 27 different sizes of capacitors. Capacitor values are often assumed as continuous variables whose costs are considered as proportional to capacitor size in past researches [14-16]. However, commercially available capacitors are discrete capacities and tuned in discrete steps. Moreover, the cost of capacitor is not linearly proportional to the size. Hence, if the continuous variable approach is used to choose integral capacitor size, the method may not result in an optimum solution and may even lead to undesirable harmonic resonance conditions [17]. However, considering all variables in a nonlinear fashion will make the placement problem very complicated. In order to simplify the analysis, only fixed-type capacitors are considered with the following assumptions: (1) balanced conditions; (2) negligible line capacitance; and (3) time-invariant loads. The objective function of this problem is to minimize f. It is composed of two parts: (1) the cost of the power loss in the transmission branch and (2) the cost of reactive power supply. Therefore, the fitness function is defined as [18].

$$f = K_p \times p_{loss} + \sum_{n=1}^{m} Q_{cj_n} K_{cj_n} \quad (5)$$

where K_p is the equivalent annual cost per unit of power loss (*$/KWatt*), n is the bus number, Kcj_n is the equiva-

ISSN 2229-5518

lent capacitor cost installed in bus n ($/KVar), $Q_{cj_n} = j \times K_S$ is the size of the capacitor, K_s is the capacitor bank size (KVar) (here K_s=150), and j= the number of banks used in any bus.

Here, a radial distribution feeder is used as an example to show the effectiveness of this algorithm. The testing distribution system is shown in Fig. 2.

This feeder has nine load buses with rated voltage of 23 kV. Tables 2 and 3 show the loads and feeder line constants. Kp is selected to be US$ 168/kW. The base value of voltage and power is 23 kV and 100 MW respectively [18]. Possible choice of capacitor sizes and costs are shown in Table 4 by assuming a life expectancy of 10 years (the placement, maintenance, and running costs are assumed to be grouped as total cost). The main procedure of finding optimal capacitor placements with FPSO method is illustrated in Fig. 3.

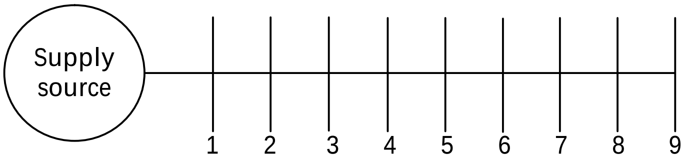

Fig. 2. Testing distribution system with nine buses

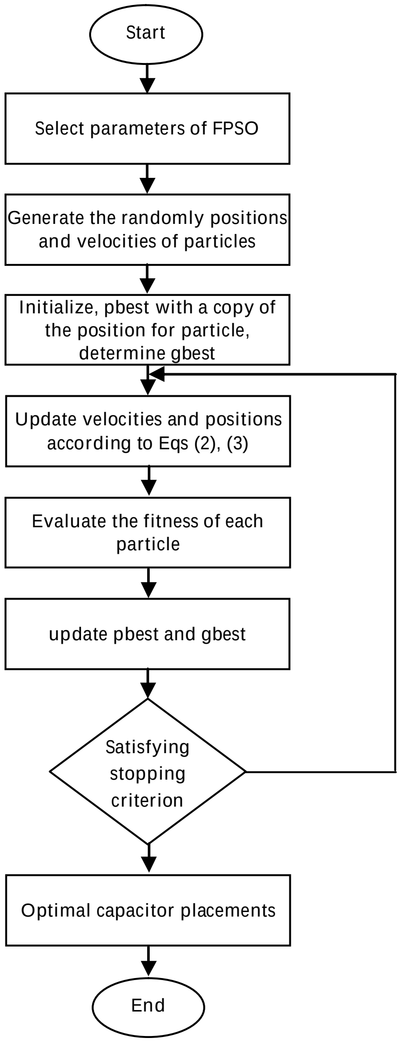

Fig. 3. Flow chart of optimal capacitor placements with FPSO

TABLE 2

LOAD DATA OF THE TEST SYSTEM

Bus	1	2	3	4	5	6	7	8	9
P(kW)	1840	980	1790	1598	1610	780	1150	980	1640
Q(Var)	460	340	446	1840	600	110	60	130	200

TABLE 3

FEEDER DATA OF THE TEST SYSTEM

From bus i	From bus j	R	X
0	1	0.1233	0.4127
1	2	0.0140	0.6051
2	3	0.7463	1.2050
3	4	0.6984	0.6084
4	5	1.9831	1.7276
5	6	0.9053	0.7886
6	7	2.0552	1.1640
7	8	4.7953	2.7160
8	9	5.3434	3.0264

TABLE 4

POSSIBLE CHOICE OF CAPACITOR SIZES AND COSTS

j	1	2	3	4	5	6	7	8	9
Q_{cj}	150	300	450	600	750	900	1050	1200	1350
K_{cj}	0.500	0.350	0.253	0.220	0.276	0.183	0.228	0.170	0.207
j	10	11	12	13	14	15	16	17	18
Q_{cj}	1500	1650	1800	1950	2100	2250	2400	2550	2700
K_{cj}	0.201	0.193	0.187	0.211	0.176	0.197	0.170	0.189	0.187
j	19	20	21	22	23	24	25	26	27
Q_{cj}	2850	3000	3150	3300	3450	3600	3750	3900	4050
K_{cj}	0.183	0.180	0.195	0.174	0.188	0.170	0.183	0.182	0.179

The effectiveness of the method is illustrated by a comparative study of the following two cases: Case 1 is without capacitor installation and Case 2 use the FPSO approach for optimizing the size and the placement of the capacitor in the radial distribution system.

The capacitor sizes, power loss and the total cost are shown in Table 5. Fig. 4 depicts the minimum value of cost function in any iteration for 100 iterations. Before optimization (Case 1), the power loss is 775 kW and total cost is 1.302e5 $. After optimization (Case 2), the power loss becomes 667.5 kW and the total cost becomes 1.130e5 $.

TABLE 5

SUMMARY RESULT OF THE APPROACH

Bus	Capacitor size (kVar) 0	1	2	3	4	5	6	7	8	9	P_{loss} (kW)	total cost ($)
Case 1	0	0	0	0	0	0	0	0	0	0	775	1.302e5
Case 2	0	1500	1500	1500	1500	400	450	300	150	300	667.5	1.130e5

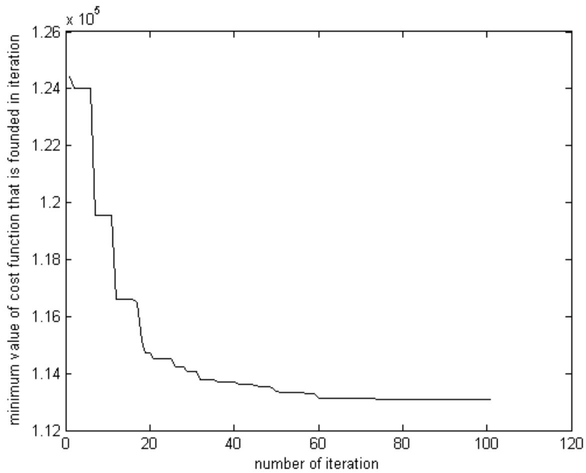

Fig. 4. Minimum value of cost function in each iteration

5 CONCLUSIONS

In this paper, a modified optimization technique based on PSO algorithm is introduced and it is called fast PSO (FPSO). Using adaptive coefficients, the step sizes of particles movements are changed appropriately to reach the optima, rapidly. The important characteristics of FPSO are: less function evaluation and high convergence rate. Consequently in real time processes, FPSO seems to outperform both the standard PSO and the genetic algorithm. The efficiency of the proposed FPSO algorithm is shown using several well-known benchmark functions. Then, the proposed FPSO method is used to successfully solve the problem of optimal capacitor selection and placement problem in radial distribution systems.

6 END SECTIONS

6.1 APPENDIX A

Some well-known benchmark functions of optimization problems [19].

Branin RCOS (*RC*) (2 variables):

$$RC(x_1,x_2)=\left(x_2-\left(\frac{5}{4\pi^2}\right)x_1^2+\left(\frac{5}{\pi}\right)x_1-6\right)^2+$$
$$10\left(1-\left(\frac{1}{8\pi}\right)\right)cos(x_1)+10;$$

Search domain: $-5 < x_1 < 10$, $0 < x_2 < 15$

no local minimum; 3 global minima

B2 (2 variables):

$$B2(x_1,x_2)=x_1^2+2x_2^2-0.3\cos(3\pi x_1)-0.4\cos(4\pi x_2)+0.7;$$

Search domain: $-100 < x_j < 100$, j=1, 2;

several local minima; 1 global minimum

Easom (*ES*) (2 variables):

$$ES(x_1,x_2)=-cos(x_1)cos(x_2)exp(-((x_1-\pi)^2+(x_2-\pi)^2));$$

Search domain: $-100<x_j<100$, j=1, 2;

several local minima, 1 global minimum

Goldstein and Price (*GP*) (2 variables):

$$GP(x_1,x_2)=$$
$$\left[\begin{array}{l}1+(+x_1+x_2+1)^2\\ \times(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)\end{array}\right]\times$$
$$\left[\begin{array}{l}30+(2x_1-3x_2)^2\times\\ (18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)\end{array}\right];$$

Search domain: $-2 < x_j < 2$, j=1, 2;

4 local minima; 1 global minimum

Shubert (*SH*) (2 variables):

$$SH(x_1,x_2)=\left\{\sum_{j=1}^{5} j\cos[(j+1)x_1+j]\right\}$$
$$\times\left\{\sum_{j=1}^{5} j\cos[(j+1)x_2+j]\right\};$$

Search domain: $-10 < x_j < 10$, j=1, 2;

760 local minima; 18 global minimum

De Joung (*DJ*) (3 variables):

$$DJ(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$$

Search domain: -5.12 < x_j < 5.12, j=1, 2, 3;
1 single minimum (local and global)

Hartmann (H3, 4) (3 variables):

$$H_{3,4}(x_1, x_2, x_3) = -\sum_{j=1}^{4} c_i \exp[-\sum_{j=1}^{3} a_{ij}(x_i - p_{ij})^2];$$

Search domain: 0 < x_j < 1, j=1, 2, 3;
4 local minima1 global minimum

Shekel ($S_{4,n}$) (3 variables):

$$S_{4,n}(X) = -\sum_{j=1}^{n} [(X - a_i)^T \times (X - a_i) + c_i]^{-1};$$

$$X = (x_1, x_2, x_3, x_4)^T; a_i = (a_i^1, a_i^2, a_i^3, a_i^4)^T;$$

Search domain: 0 < x_j < 10, j=1, 2, 3,4;
n local minima; 1 global minimum

Rosenbrock (R_n) (n variables):

$$R_n(X) = -\sum_{j=1}^{n} - [100(x_j^2 - x_{j+1})^2 + (x_j - 1)^2;$$

Search domain: -5 < x_j < 10, j=1, …, n;
several local minima;1 global minimum

Zakharov (Z_n) (n variables):

$$Z_n(X) = \left(\sum_{j=1}^{n} x_j^2\right) + (\sum_{j=1}^{n} 0.5 j x_j)^2 + (\sum_{j=1}^{n} 0.5 x_j)^4;$$

Search domain: -5 < x_j < 10, j=1, …, n;
several local minima; 1 global minimum.

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